The muon g-2: status from a theorist's point of view

Massimo Passera
INFN Padova

Physics of fundamental Symmetries and Interactions
PSI - 19 October 2016
Theory of the $g$-2: the beginning

- **Kusch and Foley 1948:**

\[
\mu_e^{\text{exp}} = \frac{e\hbar}{2mc} (1.00119 \pm 0.00005)
\]

- **Schwinger 1948 (triumph of QED!):**

\[
\mu_e^{\text{th}} = \frac{e\hbar}{2mc} \left( 1 + \frac{\alpha}{2\pi} \right) = \frac{e\hbar}{2mc} \times 1.00116
\]

- **Keep studying the lepton–$\gamma$ vertex:**

\[
\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p') \left[ \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m} F_2(q^2) + \ldots \right] u(p)
\]

\[
F_1(0) = 1 \quad F_2(0) = \alpha_l
\]

A pure “quantum correction” effect!
The muon g-2: experimental status

Today: \( a_\mu^{\text{EXP}} = (116592089 \pm 54_{\text{stat}} \pm 33_{\text{sys}}) \times 10^{-11} [0.5\text{ppm}] \).

Future: new muon g-2 experiments at:
- **Fermilab E989**: aiming at \( \pm 16 \times 10^{-11} \), ie 0.14ppm. Beam expected next year. First result expected in 2018 with a precision comparable to that of BNL E821.
- **J-PARC proposal**: aiming at 2019 Phase 1 start with 0.4ppm.

Are theorists ready for this (amazing) precision? Not yet
The muon g-2: the QED contribution

\[ a_\mu^{\text{QED}} = \frac{1}{2}(\alpha/\pi) \text{ Schwinger 1948} \]

\[ + 0.765857426 (16) (\alpha/\pi)^2 \]

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

\[ + 24.05050988 (28) (\alpha/\pi)^3 \]

Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04; Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

\[ + 130.8773 (61) (\alpha/\pi)^4 \]


\[ + 752.85 (93) (\alpha/\pi)^5 \text{ COMPLETED!} \]


Adding up, we get:

\[ a_\mu^{\text{QED}} = 116584718.941 (21)(77) \times 10^{-11} \]

from coeffs, mainly from 4-loop unc ⬢ ⬣ from δα(Rb)

with \( \alpha = 1/137.035999049(90) \) [0.66 ppb]
The muon g-2: the electroweak contribution

**One-loop term:**

\[
a_\mu^{\text{EW}}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2} \left[ 1 + \frac{1}{5} \left( 1 - 4\sin^2\theta_W \right)^2 + O\left( \frac{m_\mu^2}{M_{Z,W,H}^2} \right) \right] \approx 195 \times 10^{-11}
\]

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda; Studenikin et al. '80s

**One-loop plus higher-order terms:**

\[
a_\mu^{\text{EW}} = 153.6 (1) \times 10^{-11}
\]

with \( M_{\text{Higgs}} = 125.6 (1.5) \text{ GeV} \)

Hadronic loop uncertainties and 3-loop nonleading logs.

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013.
The muon g-2: the hadronic LO contribution (HLO)

\[ K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)(s/m^2)} \]

\[ a_{\mu}^{HLO} = \frac{1}{4\pi^3} \int_{4m_{\pi}^2}^{\infty} ds \frac{dK(s)}{ds} \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_{\pi}^2}^{\infty} ds \frac{1}{s} K(s) R(s) \]

\[ a_{\mu}^{HLO} = 6870 \ (42)_{\text{tot}} \times 10^{-11} \]

\[ = 6928 \ (33)_{\text{tot}} \times 10^{-11} \]

\[ = 6949 \ (37)_{\exp \ (21)}_{\text{rad}} \times 10^{-11} \]


Lots of progress in lattice calculations. T. Blum et al, PRL116 (2016) 232002
New space-like proposal for HLO

- Alternatively, exchanging the $x$ and $s$ integrations in $a_{\mu}^{\text{HLO}}$:

$$a_{\mu}^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx \ (1 - x) \ \Delta \alpha_{\text{had}}[t(x)] \quad t(x) = \frac{x^2 m_{\mu}^2}{x - 1} < 0$$

which involves $\Delta \alpha_{\text{had}}(t)$, the hadr. contrib. to the running of $\alpha$ in the space-like region. It can be extracted from Bhabha scattering data!
New space-like proposal for HLO (2)

- $\Delta \alpha_{\text{had}}(t)$ can also be measured via the elastic scattering $\mu e \rightarrow \mu e$.

- Scattering a beam of muons of 150 GeV, available at CERN’s North Area, on a fixed electron target, $0 < x < 0.93$ (peak at 0.91).

G. Abbiendi et al, arXiv:1609.08987

- With CERN’s 150 GeV muon beam ($1.3 \times 10^7 \mu/s$ average) a statistical uncertainty of $\sim 0.3\%$ ($\sim 20 \times 10^{-11}$) can be reached on $a_\mu^{\text{HLO}}$ with 2 years of data taking. 10ppm systematic accuracy needed at peak.
**HNLO: Vacuum Polarization**

\[ a_{\mu}^{\text{HNLO}(\text{vp})} = -98 \pm 1 \times 10^{-11} \]

\( \mathcal{O}(\alpha^3) \) contributions of diagrams containing hadronic vacuum polarization insertions:

Krause '96, Alemany et al. '98, Hagiwara et al. 2011
**HNLO: Light-by-light contribution**

Unlike the HLO term, the hadronic l-b-l term relies at present on theoretical approaches.

This term had a troubled life! Latest values:

\[
a_{\mu}^{\text{HNLO}}(\text{lbl}) = \begin{align*}
+80 \ (40) \times 10^{-11} & \quad \text{Knecht & Nyffeler '02} \\
+136 \ (25) \times 10^{-11} & \quad \text{Melnikov & Vainshtein '03} \\
+105 \ (26) \times 10^{-11} & \quad \text{Prades, de Rafael, Vainshtein '09} \\
+102 \ (39) \times 10^{-11} & \quad \text{Jegerlehner, arXiv:1511.04473}
\end{align*}
\]

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02

**Improvements expected in the \( \pi^0 \) transition form factor** A. Nyffeler 1602.03398


**Progress on the lattice:** +53.5(13.5)x10^{-11}. Statistical error only, finite-volume and finite lattice-spacing errors being studied. Omitted subleading disconnected graphs still need to be computed.

The muon g-2: the hadronic NNLO contributions (HNNLO)

- **HNNLO: Vacuum Polarization**
  
  O($\alpha^4$) contributions of diagrams containing hadronic vacuum polarization insertions:

  \[ a_\mu^{\text{HNNLO}(\text{vp})} = 12.4 (1) \times 10^{-11} \]

  Kurz, Liu, Marquard, Steinhauser 2014

- **HNNLO: Light-by-light**
  
  \[ a_\mu^{\text{HNNLO}(\text{lbl})} = 3 (2) \times 10^{-11} \]

  Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014
The muon g-2: SM vs. Experiment

Comparisons of the SM predictions with the measured g-2 value:

\[ a_{\mu}^{\text{EXP}} = 116592091 (63) \times 10^{-11} \]

<table>
<thead>
<tr>
<th>( a_{\mu}^{\text{SM}} \times 10^{11} )</th>
<th>( \Delta a_{\mu} = a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}} )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>116 591 761 (57)</td>
<td>330 (85) \times 10^{-11}</td>
<td>3.9 [1]</td>
</tr>
<tr>
<td>116 591 820 (51)</td>
<td>271 (81) \times 10^{-11}</td>
<td>3.3 [2]</td>
</tr>
<tr>
<td>116 591 841 (58)</td>
<td>250 (86) \times 10^{-11}</td>
<td>2.9 [3]</td>
</tr>
</tbody>
</table>

with the recent “conservative” hadronic light-by-light \( a_{\mu}^{\text{HNLO(lbl)}} = 102 (39) \times 10^{-11} \) of F. Jegerlehner arXiv:1511.04473, and the hadronic leading-order of:

Brief digression: the electron g-2
The electron g-2: SM vs Experiment

- The 2008 measurement of the electron g-2 is:

\[ a_e^{\text{EXP}} = 11596521807.3 \pm 2.8 \times 10^{-13} \]

Hanneke, Fogwell, Gabrielse
PRL100 (2008) 120801

- Using \( \alpha = 1/137.035999049 \pm 0.000090 \) from h/M measurement of \( ^{87}\text{Rb} \) (2011), the SM prediction for the electron g-2 is

\[ a_e^{\text{SM}} = 11596521816.5 \pm 0.2 \text{ (0.2) (0.2) (7.6) } x 10^{-13} \]

- The EXP-SM difference is (note the negative sign):

\[ \Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}} = -9.2 \pm 8.1 \times 10^{-13} \]

The SM is in very good agreement with experiment (1\( \sigma \)).
The electron g-2 sensitivity and NP tests

- The present sensitivity is \( \delta \Delta a_e = 8.1 \times 10^{-13} \), i.e. (10^{-13} units):

\[
(0.2)_{\text{QED4}}, \quad (0.2)_{\text{QED5}}, \quad (0.2)_{\text{HAD}}, \quad (7.6)_{\delta \alpha}, \quad (2.8)_{\delta a_e^{\text{EXP}}}
\]

\[
(0.4)_{\text{TH}} \quad \text{may drop to 0.2}
\]

- The \((g-2)_e\) exp. error may soon drop below \(10^{-13}\) and work is in progress for a significant reduction of that induced by \(\delta \alpha\).

\( \rightarrow \) sensitivity of \(10^{-13}\) may be reached with ongoing exp. work

- In a broad class of BSM theories, contributions to \(a_\perp\) scale as

\[
\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}}\right)^2
\]

This Naive Scaling leads to:

\[
\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}
\]
The experimental sensitivity in $\Delta a_e$ is not too far from what is needed to test if the discrepancy in $(g-2)_\mu$ also manifests itself in $(g-2)_e$ under the naive scaling hypothesis.

NP scenarios exist which violate Naive Scaling. They can lead to larger effects in $\Delta a_e$ and contributions to EDMs, LFV or lepton universality breaking observables.

Example: In the MSSM with non-degenerate but aligned sleptons (vanishing flavor mixing angles), $\Delta a_e$ can reach $10^{-12}$ (at the limit of the present exp sensitivity). For these values one typically has breaking effects of lepton universality at the few per mil level (within future exp reach).

Giudice, Paradisi, MP  JHEP 2012
Back to the muon g-2
Δa_μ: could it be errors in the hadronic cross section?

- Can Δa_μ be due to hypothetical mistakes in the hadronic σ(s)?
- An upward shift of σ(s) also induces an increase of Δα_{had}^{(5)}(M_Z).
- Consider:

\[
\begin{align*}
\text{a}_{μ}^{\text{HLO}} & \rightarrow \quad a = \int_{4m_{π}^{2}}^{s_u} ds \, f(s) \, σ(s), \quad f(s) = \frac{K(s)}{4π^3}, \quad s_u < M_Z^2; \\
\text{Δα}_{\text{had}}^{(5)} & \rightarrow \quad b = \int_{4m_{π}^{2}}^{s_u} ds \, g(s) \, σ(s), \quad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4απ^2)};
\end{align*}
\]

and the increase

\[Δσ(s) = εσ(s)\]

(ε>0), in the range:

\[\sqrt{s} \in [\sqrt{s_0} - δ/2, \sqrt{s_0} + δ/2]\]
How much does the $M_H$ upper bound from the EW fit change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$] to accommodate $\Delta a_\mu$?
Given the quoted exp. uncertainty of $\sigma(s)$, the possibility to explain the muon g-2 with these very large shifts $\Delta\sigma(s)$ appears to be very unlikely.

Also, given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energy (below ~ 1 GeV) where $\sigma(s)$ is precisely measured.

Vice versa, assuming we now have a SM Higgs with $M_H = 125$ GeV, if we bridge the $M_H$ discrepancy in the EW fit decreasing the low-energy hadronic cross section, the muon g-2 discrepancy increases.

W.J. Marciano, A. Sirlin, MP, 2008 & 2010
ALPs contributions to the muon g-2

Light spin 0 scalars & pseudoscalars (axion-like-particles or ALPs), contribute to $a_\mu$. We consider ALPs in the mass range \([0.1–1]\ \text{GeV}\), where experimental constraints are rather loose.

A possible resolution of $\Delta a_\mu$ by 1-loop contributions from scalar particles with relatively large Yukawa couplings to muons, of $O(10^{-3})$, was analyzed by Chen, Davoudiasl, Marciano & Zhang, PRD 93, 035006 (2016):

For a pseudoscalar, the 1-loop contribution has the wrong sign (negative) to resolve the discrepancy on its own.
Consider ALP-γγ couplings as well as Yukawa couplings:

\[
\mathcal{L}_a = \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu} \tilde{F}^{\mu\nu} + i y_{a\psi} a \bar{\psi} \gamma_5 \psi,
\]

\[
\mathcal{L}_s = \frac{1}{4} g_{s\gamma\gamma} s F_{\mu\nu} F^{\mu\nu} + y_{s\psi} s \bar{\psi} \psi
\]

New, potentially important, ALP contributions to \(a_\mu\):

ALPs contributions to the muon g-2 (3)

\[
\begin{align*}
Y &\quad a_{\ell,a}^Y < 0 \\
BZ &\quad a_{\ell,a}^{BZ} \simeq \left( \frac{m_\ell}{4\pi^2} \right) g_{a\gamma\gamma} y_{a\ell} \ln \frac{\Lambda}{m_a} \\
LbL &\quad a_{\ell,a}^{LbL} \simeq 3 \frac{\alpha}{\pi} \left( \frac{m_\ell g_{a\gamma\gamma}}{4\pi} \right)^2 \ln^2 \frac{\Lambda}{m_a} > 0 \\
VP &\quad a_{\ell,a}^{VP} \simeq \frac{\alpha}{\pi} \left( \frac{m_\ell g_{a\gamma\gamma}}{12\pi} \right)^2 \ln \frac{\Lambda}{m_a} > 0
\end{align*}
\]

- **For a scalar ALP**, change the signs of \( Y \) & \( LbL \).
- **The sign of BZ** depends on the couplings. We assume it’s > 0.
- **VP** is positive both for scalar & pseudoscalar, but negligible.

Both pseudoscalar and scalar ALPs can solve $\Delta a_\mu$ for masses and couplings allowed by current exp. constraints.

They can be tested at present low-energy $e^+e^-$ colliders through dedicated $e^+e^- \rightarrow e^+e^- + \text{ALP}$ searches.
Conclusions

The lepton g-2 provide beautiful examples of interplay between theory and experiment, and different areas of Physics.

**Muon g-2:** $\Delta a_\mu \sim 3.5 \sigma$. New upcoming experiment: QED & EW ready. Lots of progress in the hadr sector, but not yet ready!

New proposal to measure the leading hadronic contribution to the muon g-2 via $\mu$-e elastic scattering at CERN.

**Electron g-2:** Does the discrepancy in $(g-2)_\mu$ also manifests itself in $(g-2)_e$? NP sensitivity limited by exp. uncertainties, but a strong exp. program is under way to improve both $\alpha$ & $a_e$.

Could $\Delta a_\mu$ be due to mistakes in the hadronic $\sigma(s)$? Very unlikely. Also, given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energies ($\lesssim 1$GeV).

Light spin 0 scalars & pseudoscalars can solve $\Delta a_\mu$ for masses and couplings allowed by current experimental bounds. Dedicated searches can test them at low-energy $e^+e^-$ colliders.
The End
Backup
The QED prediction of the electron g-2

\[ a_e^{\text{QED}} = + \frac{1}{2} \left( \frac{\alpha}{\pi} \right) - 0.32847844400255(33) \left( \frac{\alpha}{\pi} \right)^2 \]

Schwinger 1948  Sommerfeld; Petermann; Suura&Wichmann ’57; Elend ’66; CODATA Mar ’12

\[ A_1^{(4)} = -0.32847896557919378... \]
\[ A_2^{(4)} \left( \frac{m_e}{m_\mu} \right) = 5.19738668(26) \times 10^{-7} \]
\[ A_2^{(4)} \left( \frac{m_e}{m_\tau} \right) = 1.83798(33) \times 10^{-9} \]

+ 1.181234 016816 (11) \left( \frac{\alpha}{\pi} \right)^3

Kinoshita; Barbieri; Laporta, Remiddi; … , Li, Samuel; MP ’06; Giudice, Paradisi, MP 2012

\[ A_1^{(6)} = 1.181241456587... \]
\[ A_2^{(6)} \left( \frac{m_e}{m_\mu} \right) = -7.37394162(27) \times 10^{-6} \]
\[ A_2^{(6)} \left( \frac{m_e}{m_\tau} \right) = -6.5830(11) \times 10^{-8} \]
\[ A_3^{(6)} \left( \frac{m_e}{m_\mu}, \frac{m_e}{m_\tau} \right) = 1.90982(34) \times 10^{-13} \]

- 1.91206 (84) \left( \frac{\alpha}{\pi} \right)^4


+ 7.79 (34) \left( \frac{\alpha}{\pi} \right)^5  \text{ Complete Result! (12672 mass indep. diagrams!)}


NB: \left( \frac{\alpha}{\pi} \right)^6 \sim O(10^{-16})
The SM prediction of the electron g-2

The SM prediction is:

\[ a_{e}^{\text{SM}} (\alpha) = a_{e}^{\text{QED}} (\alpha) + a_{e}^{\text{EW}} + a_{e}^{\text{HAD}} \]

The EW (1&2 loop) term is:

Czarnecki, Krause, Marciano ’96 [value from Codata10]

\[ a_{e}^{\text{EW}} = 0.2973 (52) \times 10^{-13} \]

The Hadronic contribution, at LO+NLO+NNLO, is:

Nomura & Teubner ’12, Jegerlehner & Nyffeler ’09; Krause’97; Kurz, Liu, Marquard & Steinhauser 2014

\[ a_{e}^{\text{HAD}} = 17.10 (17) \times 10^{-13} \]

\[ a_{e}^{\text{HLO}} = + 18.66 (11) \times 10^{-13} \]

\[ a_{e}^{\text{HNLO}} = [-2.234 (14)_{\text{vac}} + 0.39 (13)_{\text{lbl}}] \times 10^{-13} \]

\[ a_{e}^{\text{HNNLO}} = + 0.28 (1) \times 10^{-13} \]

Which value of \( \alpha \) should we use to compute \( a_{e}^{\text{SM}} \)?
The electron g-2 gives the best determination of alpha

- The 2008 measurement of the electron g-2 is:
  \[ \alpha_{e}^{\text{EXP}} = 11596521807.3 \pm 7 \times 10^{-13} \]  
  Hanneke et al, PRL100 (2008) 120801

  vs. old (factor of 15 improvement, 1.8σ difference):
  \[ \alpha_{e}^{\text{EXP}} = 11596521883 \pm 42 \times 10^{-13} \]  
  Van Dyck et al, PRL59 (1987) 26

- Equate \( \alpha_{e}^{\text{SM}}(\alpha) = \alpha_{e}^{\text{EXP}} \) → best determination of alpha:
  \[ \alpha^{-1} = 137.035999158 \pm 33 \times 10^{-9} \]  
  [0.24 ppb]

- Compare it with other determinations (independent of \( \alpha_{e} \)):
  \[ \alpha^{-1} = 137.0360000 (11) \]  
  [7.7 ppb] PRA73 (2006) 032504 (Cs)
  \[ \alpha^{-1} = 137.035999049 (90) \]  
  [0.66 ppb] PRL106 (2011) 080801 (Rb)

Excellent agreement → beautiful test of QED at 4-loop level!