**TRIUMF**

- TRIUMF is Canada’s national laboratory for particle and nuclear physics. It is located in Vancouver, BC.
- An ultra-cold neutron (UCN) source and neutron electric dipole moment (nEDM) experiment are currently under construction at TRIUMF.

**The nEDM measurement with Ramsey’s method of separated oscillatory fields**

- Neutrons enter a cylindrical storage cell polarized in the positive z direction (vertically).
- Two static fields are present: A magnetic field acting in the positive z direction ($B_z$), and an electric field acting parallel or anti-parallel to $B_z$ (E Parallel or anti-Parallel).
- Since we know $\omega = -2\gamma B_z = -2\delta$, by running the simulation with $E_z$, then with $B_z$, we can determine the EDM.
- A rotating field, $B_z$, is applied in the xy plane at frequency $\omega_0$ for a time $\tau$. This will flip the spin by some amount.
- The neutrons are then allowed to precess freely for a time $T$. After this, the $B_z$ field is applied again for a time $\tau$.
- The number of spin up and spin down neutrons are counted.
- This is repeated for 4 different $\omega_0$ values (as in the graph below), and done for both $E_z$ and $E_{xy}$.
- $\omega_0$ is calculated between $E_z$ and $E_{xy}$. The EDM can then be determined: $d_e = \frac{\omega_0}{4\gamma}$.
- A non-zero value of the nEDM has not yet been discovered. Currently we know $|d_e| < 2.9 \times 10^{-26}$ ecm [2].

**PENTrack**

- PENTrack is a free Monte Carlo program for tracking Protons, Electrons, and Neutrons.
- Recent changes make it possible to run full simulations of Ramsey’s method of separated oscillatory fields, and track the spin of Neutrons, Xenon, and Mercury.
- EDM simulator Features:
  - Analytical and/or numerical $B_z$, $B_{xy}$, and E fields.
  - Spin tracking, Larmor frequency tracking, simultaneous $E_z$ and $E_{xy}$.
  - Compensation for EvE effect.
  - Geometry import from .sl file (Ability to design with most 3D CAD programs).

**Effect of Magnetic Field Gradients on Larmor frequency**

- The Larmor frequency is close to $\omega_0 = -\gamma B_z$; however, there are some factors that will shift it from this value.
- These shifts need to be accounted for in order to achieve the high precision required in the nEDM experiment.
- Magnetic field inhomogeneities account for a significant portion of the systematic error in the most accurate nEDM experiment [2].
- Here, the effect of magnetic field inhomogeneities on the Larmor frequency are simulated by running Ramsey cycles similar to the real experiment.
- Parameters used: $B_z = 1 \text{ mT}$, $B_{xy} = \pi$, $B_{xy} = B_z/2$, $T = 50 \text{ s}$, $\tau = 1 \text{ s}$, $E = 0$.
- $B_{xy}$ denotes a linearly oscillating $B_z$ field, and $B_{xy}$ denotes a circularly oscillating $B_z$ field.
- The neutron energy distribution is taken from a filling simulation for TRIUMF’s Phase 2 nEDM experiment.
- EDM cell height $H = 14 \text{ cm}$, radius $R = 18.1 \text{ cm}$.
- The $\omega_0$ values shown are calculated assuming $E = 1 \times 10^8 \text{ V/m}$ (Proposed electric field strength of TRIUMF’s Phase 2 nEDM experiment).

<table>
<thead>
<tr>
<th>$\omega_0$ (Hz)</th>
<th>$\omega_0$ (Hz)</th>
<th>$\omega_0$ (Hz)</th>
<th>$\omega_0$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-29.164695302$</td>
<td>$-29.165092779$</td>
<td>$-29.166077779$</td>
<td>$-29.167062779$</td>
</tr>
</tbody>
</table>

**Mathematical Section**

- Larmor frequency shift due to average UCN height in the EDM cell [5]
  $$\omega_0 = \omega_0 + \omega_0 \gamma B \beta$$
  (1)
  - $\omega_0$ is the neutron’s gyromagnetic ratio, $\beta$ is the average UCN height, and $\gamma$ is the gyromagnetic ratio of the cell, $g$, and is gravity, and $H$ is the height of the cell. The right hand of Equation 2 applies for neutrons that can reach the top of the cell (and assuming specular reflection).
- The Becch庭-Segn (BS) shift [5]
  $$\omega_0 = \omega_0 \gamma B \beta$$
  (3)
  - The BS shift applies if the B field is oscillating in 1 dimension only ($B_{xy}$).
- The Geometric Phase Effect (GPE) [6]
  $$\omega_0 = \omega_0 \gamma B \beta$$
  (4)
  - $\gamma$ is the speed of light, $\omega_0$ is the velocity in the xy plane, $H$ is the radius of the storage cell and $\gamma$ is the ratio of the path in the y direction, and $\gamma$ is the ratio of the path in the x direction. Equation 5 applies when there is polarized reflection, and Equation 7 applies when there is specular reflection.

<table>
<thead>
<tr>
<th>$\omega_0$ (Hz)</th>
<th>$\omega_0$ (Hz)</th>
<th>$\omega_0$ (Hz)</th>
<th>$\omega_0$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-29.164695302$</td>
<td>$-29.165092779$</td>
<td>$-29.166077779$</td>
<td>$-29.167062779$</td>
</tr>
</tbody>
</table>

**Geometric Phase Effect**

- The GPE will create a false EDM signal in the experiment.
- The parameters and procedure of this simulation are similar to the simulation above.
- Simultaneous $E_z$ and $E_{xy}$ spin tracking is used (Therefore, no is expected).
- Only one neutron velocity is used.
- $E = 1 \times 10^8 \text{ V/m}$ (to magnify effect).
- Some possible reasons for discrepancies are resolution/accuracy issues in PENTrack, or that Equations 5 and 7 are not valid for this situation.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Specular reflection</th>
<th>Partly diffuse reflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_0$</td>
<td>$\omega_0$</td>
<td>$\omega_0$</td>
</tr>
<tr>
<td>$3 \text{ m/s}$</td>
<td>$3 \text{ m/s}$</td>
<td>$3 \text{ m/s}$</td>
</tr>
<tr>
<td>$2.78 \times 10^{-22} \text{ ecm}$</td>
<td>$2.78 \times 10^{-22} \text{ ecm}$</td>
<td>$2.78 \times 10^{-22} \text{ ecm}$</td>
</tr>
<tr>
<td>$2.78 \times 10^{-22} \text{ ecm}$</td>
<td>$2.78 \times 10^{-22} \text{ ecm}$</td>
<td>$2.78 \times 10^{-22} \text{ ecm}$</td>
</tr>
<tr>
<td>$2 \times 10^{-20} \text{ ecm}$</td>
<td>$2 \times 10^{-20} \text{ ecm}$</td>
<td>$2 \times 10^{-20} \text{ ecm}$</td>
</tr>
</tbody>
</table>